Group actions with Applications in Geometry and Analysis

Reims, June 3-6, 2013

Yves BENOIST : « Tempered homogeneous spaces »

The regular representation in a complex reductive homogeneous space G/H is tempered if and only if the stabilizer in H of a generic point on G/H is abelian.

Jean-Louis CLERC : « Intertwining operators on symmetric R-spaces »

Let G be a real semi-simple Lie group, P a parabolic subspace and K a maximal compact subgroup associated to a Cartan involution σ of G. Then $X = G/P$ is a *symmetric R-space* if X viewed as $X \simeq K/(K \cap P)$ is a (compact Riemannian) symmetric space. I will recall the one-to-one correspondance (due to O. Loos) between symmetric R-spaces and *positive Jordan triple systems (PJTS)*. The natural action of G on the spaces of densities on X gives raise to a one complex parameter family of representations π_λ of G. From the general theory of Knapp-Stein, there is (generically in the parameter λ) an intertwining operator between π_λ and $\pi^\sigma_{-\lambda}$, where $\pi^\sigma_\mu = \pi_\mu \circ \sigma$. An explicit analytic expression (in terms of the associated PJTS) is obtained for the intertwining operator. The domain of convergence of the corresponding integrals will be discussed. The meromorphic continuation in the parameter λ will be achieved by means of a *Bernstein-Sato identity*, with an explicit description of the poles.

Michel DUFLO : « Box splines and Kirillov's formulas »

Deconvolution of Box splines, together with Kirillov's type character formulas, may be used to prove some cases of Guillemein-Sternberg conjecture.

This is joint work with Michèle Vergne.

Sigurdur HELGASON : « Eigenspaces and Eigenspace Representations »

On a coset space G/K let $D(G/K)$ denote the algebra of differential operators which are invariant under the action of G. For a homomorphism h of $D(G/K)$ into $\mathbb{C}$ let $E(h)$ denote the corresponding joint eigenspace consisting of functions f satisfying

$$Df = h(D)f \quad \text{for all } D \in D(G/K).$$

and let $T(h)$ denote the natural representation of G on $E(h)$. We shall discuss results and problems for examples of the explicit determination of $E(h)$ as well as results about the representations $T(h)$.

Roger HOWE : « Structure of holomorphic unitary representations »

Within the broad area of representation theory of reductive groups, holomorphic unitary representations have been an object of particular study, both because they are relatively accessible, and because they provide connections between group theory and complex analysis and geometry. Most holomorphic unitary representations belong to the discrete series, and were the object of Harish-Chandra's early efforts toward a general Plancherel formula for semi simple groups. They have a fairly uniform and well-understood structure. On the other hand, the structure of the more singular holomorphic representations has not been described very explicitly. This talk will discuss an approach to understanding singular holomorphic representations by means of a refinement of classical invariant theory.

Fanny KASSEL : « Spectral analysis on non-Riemannian locally symmetric spaces »

At the end of the 1980's, Toshiyuki Kobayashi initiated the systematic study of properly discontinuous actions on reductive homogeneous spaces G/H with H noncompact. Of particular importance are properly discontinuous actions on reductive symmetric spaces. The corresponding quotients are naturally endowed with a Laplacian ; this differential operator is not elliptic in the non-Riemannian case. Kobayashi and I prove that its discrete spectrum is nonempty, and even infinite, as soon as the locally symmetric space is "sharp", i.e. defined by a "sufficiently proper" action. We also investigate the behavior of the spectrum under small deformations, and exhibit some rigidity phenomenon that has no counterpart in the Riemannian world.

Alexander KIRILLOV : « Generalized exponents and family algebras »

To any irreducible representation π_λ of a simple Lie algebra $\mathfrak{g}$ the set of generalized exponents $e_1(\lambda), \ldots, e_k(\lambda)$ is defined. It is the so-called q-analogue of the multiplicity k of the zero weight. The search of a simple and effective way to compute these exponents is a part of the evergreen problem of multiplicities of weights in representation theory.

Working on this problem, I defined a new class of associative algebras which I called classical and quantum family algebras. In my talk I list known facts and new results about generalized exponents which can be found or interpreted using family algebras. A variant of this talk I gave last summer on the conference in memory of J.-M. Souriau in Aix-en Provence.

Toshiyuki KOBAYASHI : « Branching, Multiplicities and real spherical varieties »

Branching problems ask how irreducible representations of groups decompose when restricted to subgroups.

Decompositions of tensor product representations, Littlewood–Richardson's rules, and Blattner formulæ are classical examples of branching laws for symmetric pairs.

However, we observe that bad features like "infinite multiplicities" may well happen in dealing with branching problems of irreducible representations of real reductive groups G when restricted to maximal reductive subgroups G', even if (G, G') are symmetric pairs.

In this talk I plan to discuss what is a "nice framework" in which we could expect to develop a fruitful and detailed analysis on branching laws. In connection with the theory of "real spherical varieties", I plan to give a classification of reductive symmetric pairs (G, G') for which multiplicities are always finite or bounded.

Toshihisa KUBO : « Constructions of explicit homomorphisms between generalized Verma modules »

In this short talk we will discuss two different constructions of explicit homomorphisms between generalized Verma modules (equivalently, constructions of explicit covariant differential operators between homogeneous vector bundles). One of them is based on the so-called "F-method" recently developed by T. Kobayashi and his collaborators.

Jan MÖLLERS : « Model intertwining operators and applications to automorphic forms »

We explicitly construct intertwining operators between principal series representations of a semisimple group G and principal series of a symmetric subgroup $H \subseteq G$. These intertwiners generalize both the classical Knapp-Stein intertwiners and the intertwiners coming from invariant trilinear forms on principal series. The new intertwiners are given in terms of their integral kernels and we study convergence of the integrals, meromorphic continuation and uniqueness properties. As an application we present estimates for the restriction of automorphic forms on hyperbolic manifolds to hyperbolic submanifolds. This is joint work with Y. Oshima and B. Ørsted.

Salma NASRIN : « Corwin–Greenleaf multiplicity function and Kobayashi's multiplicity-free theorem »

The Kirillov–Kostant–Duflo orbit philosophy suggests that there is a closely relation between unitary representations of Lie groups and the geometry of coadjoint orbits, although it is known that the correspondence does not work perfectly for reductive groups. In this talk I plan to discuss the 'classical limit' for some special cases of Kobayashi's multiplicity-free theorems (based on visible actions) and his discrete decomposability theorems.

Karl-Hermann NEEB : « A representation theoretic perspective on reflection positivity »

Reflection positivity (sometimes called Osterwalder-Schrader positivity) was introduced by Osterwalder and Schrader in the context of axiomatic euclidean field theories. On the level of unitary representations, it provides a passage from representations of the euclidean isometry group to representations of the Poincaré group. In our talk we shall explain how these ideas can be used to obtain a natural context for the passage from representations of symmetric Lie groups to representations of their dual Lie group. In particular, we shall discuss the role of distribution vectors in this picture and how distributions on a Lie group can lead to “reflection positive“ representations.

Takayuki OKUDA : « Kobayashi's properness criterion and its applications »

In 1962, E. Calabi and L. Markus showed that no infinite discrete subgroup of $SO_0(n+1,1)$ acts properly discontinuously on the de Sitter space $SO_0(n+1,1)/SO_0(n,1)$ [Ann. Math. 1962]. More generally, if a homogeneous space G/H does not admit any infinite discontinuous group, we say that a Calabi–Markus phenomenon occurs for G/H. In 1989, T. Kobayashi proved that for homogeneous spaces G/H of reductive type, the Calabi–Markus phenomenon occurs if and only if the real rank of H is equal to that of G [Math. Ann. 1989]. To this, he made a criterion for the properness of an action on G/H of a given closed subgroup L of G. In this talk, we give a complete classification of irreducible symmetric spaces G/H for which there exist proper $SL(2,\mathbb{R})$-actions as isometries, using Kobayashi's properness criterion and combinatorial techniques of nilpotent orbits. In particular, we classify irreducible symmetric spaces that admit surface groups as discontinuous groups, combining this with Y. Benoist's results [Ann. Math. 1996].

Yoshiki OSHIMA : « Discrete branching laws of Zuckerman's derived functor modules »

The Zuckerman functor provides a certain class of unitary representations of real reductive Lie groups, known as $A_{\mathfrak{q}}(\lambda)$. In this talk, we study branching laws of $A_{\mathfrak{q}}(\lambda)$ for semisimple symmetric pairs. T. Kobayashi advocated the study of branching problems, in particular, by introducing a nice framework of problems such as the notion of "admissible restriction". In this framework we obtain explicit branching formulas by using D-modules on the flag variety.

V. Souček : « The BGG resolutions in singular character »

The construction of the BGG resolution for a finite dimensional representations of a simple Lie algebra in terms of generalized Verma modules is a classical and well known result. It can be formulated either in terms of homomorphisms between generalized Verma modules or (dually) using intertwining differential operators between the corresponding spaces of sections of homogeneous vector bundles over flag manifolds. Similar resolutions in singular infinitesimal character are much less understood.

There are different motivations for interest in a better understanding of the singular case. Apart from interest coming from representation theory itself (classification of Kostant modules), these resolutions play an important role also in differential geometry (construction of invariant different operators, curved analogues of the BGG complexes) as well as in hypercomplex analysis (analogues of the Dolbeault resolutions from several complex variables).

In the lecture, we shall describe various geometrical constructions of the BGG resolutions in case of singular infinitesimal character and their applications in differential geometry and complex analysis.

Birgit SPEH : « Intertwining operators for rank one orthogonal groups »

Intertwining operators for principal series representations were introduced by F. Bruhat and later studied in detail by A.W. Knapp and E. Stein. In this lecture I will discuss intertwining operators between spherical principal series representations of the orthogonal group O(n,1) and the subgroup O(n-1,1). This is joint work with T. Kobayashi.

Aleksander STRASBURGER : « Spherical harmonic expansions from differentiable viewpoint »

We shall present an account of a recently developed method of spherical harmonic expansions of smooth zonal function defined on Euclidean spheres, with particular empahsis on the case of the unit sphere in $\mathbb{C}^n$. In contrast to the classical method, the coefficients of the expansion are expressed in terms of the Taylor coefficients of the zonal function. Time permitting, a complex form of the Funk-Hecke formula will be derived as a corollary. The presentation is based on a joint work with A. Bezubik.

Aleksy TRALLE : « On simply-connected K-contact non-Sasakian manifolds »

We solve the problem posed by Boyer and Galicki about the existence of simply-connected K-contact manifolds with no Sasakian structure. We prove that such manifolds do exist using the method of fat bundles developed in the framework of symplectic and contact geometry by Sternberg, Weinstein and Lerman. Our techniques essentially use constructions expressed in terms of Lie group actions.

T.N. VENKATARAMANA : « Monodromy and arithmetic groups »

We consider d-fold cyclic coverings of the projective line over C which ramify over $n+1$ points in the complex plane. If the ramification degrees are all one, then we show that for $n \geq 2d$, the monodromy action (as the branch points vary) on the first cohomology of the cyclic covering is an arithmetic groups. [This is in contrast to the result of Deligne and Mostow, who exhibit non-arithmetic monodromy for small d and small n].

Joseph WOLF : « Stepwise Square Integrable Representations of Nilpotent Lie Groups »

We study the conditions for a nilpotent Lie group to be foliated into subgroups that have square integrable (relative discrete series) unitary representations, that fit together to form a filtration by normal subgroups.

Then we use that filtration to construct a class of "stepwise square integrable" representations on which Plancherel measure is concentrated. Further, we work out the character formulae for those stepwise square integrable representations, and we give an explicit Plancherel formula. Next, we use some structure theory to check that all these constructions and results apply to nilradicals of minimal parabolic subgroups of real reductive Lie groups. Then we develop multiplicity formulae for compact quotients N/Γ where Γ respects the filtration. Finally, if time permits, we look at the Plancherel formula for minimal parabolic subgroups.

Taro YOSHINO : « On Topological Blow-up »

Consider a Lie group G (or more generally, a topological group) acts continuously on a manifold M (or more generally, a locally compact Hausdorff space). The quotient space $X := G\backslash H$ is locally compact, but not always Hausdorff. In this talk, we introduce a method to understand the topology on such a non-Hausdorff space X. More precisely, for a given locally compact (not necessarily Hausdorff) space X, we construct a locally compact Hausdorff space Y, and a map $\tau : X \to 2^X$. Then, the pair (Y, τ) has a complete information on the topology on X. In particular, (Y, τ) describes convergence of sequences or filters on X.

Genkai ZHANG : « Branching of complementary series representations of rank one groups »

We consider the following two problems on branching rules : (a) Restriction of a complementary series of rank one groups to symmetric subgroups, (b) Tensor products of complementary series of rank one groups. I shall give a survey of recent results on the solution of the two problems. Some parts are joint work with B. Speh and some are work in progress with J. Möllers and B. Ørsted.