# 42$^d$ SOPHUS LIE SEMINAR AND LMNRS MEETING ON REPRESENTATION THEORY AND HARMONIC ANALYSIS

REIMS MATHEMATICS LAB, CAMPUS MOULIN DE LA HOUSSE, BDG 3, AMPHI E

## THURSDAY, FEBRUARY 9.

14h00-15h00. Jacques FARAUT, "*Harmonic analysis on Olshanski spherical pairs*".

Abstract: An Olshanski spherical pair $(G, K)$ is the inductive limit of an increasing sequence $(G_n, K_n)$ of Gelfand pairs:

$$G = \bigcup_{n=1}^{\infty} G_n, \qquad K \bigcup_{n=1}^{\infty} K_n.$$

The spherical dual $\Omega$ of the Olshanski spherical pair $(G, K)$ can be identified to the set of spherical functions of positive type. An analogue of the Bochner-Godement theorem has been obtained by Rabaoui: for a continuous function $\varphi$ of positive type on $G$, which is $K$-biinvariant, there is a unique positive measure $\mu$ on $\Omega$ such that

$$\varphi(x) = \int_{\Omega} \varphi(\omega, x)\mu(d\omega).$$

The proof uses Choquet's theory of integral representation in convex cones.

J. Faraut, Asymptotic spherical analysis, Contemporary Math. 2011.

M. Rabaoui, A Bochner type theorem for inductive limits of Gelfand pairs, Ann. Institut Fourier 2008.

15h00-16h00. Salah MEHDI. "*Translation principle for Dirac cohomology and representation theoretic harmonic spinors.*"

Abstract: We will describe several results relating harmonic spinors and representations of real reductive Lie groups. In particular we will describe the behavior of the Dirac cohomology of (g,K)-modules under the Jantzen-Zuckerman translation functors.

16h30-17h30. Helge GLÖCKNER. " *Convolution of test functions on Lie groups*".

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Abstract: Surprisingly, it has only been shown in recent years that the bilinear map

$$C_c^\infty(\mathbb{R}^n) \times C_c^\infty(\mathbb{R}^n) \to C_c^\infty(\mathbb{R}^n)$$

taking a pair $(f, g)$ of test functions to their convolution $f * g$ is continuous (Hirai et al. 2001).

The talk is devoted to corresponding questions for a Lie group $G$. I first explain that the convolution map

$$C_c^\infty(G) \times C_c^\infty(G) \to C_c^\infty(G)$$

is continuous if and only if $G$ is $\sigma$-compact.

More generally, let $E_1, E_2$ and $F$ be locally convex spaces such that $F$ is sequentially complete, and $b : E_1 \times E_2 \to F$ be a non-zero continuous bilinear map. Let $r, s, t \in \mathbb{N}_0 \cup \{\infty\}$ such that $t \leq r + s$ and

$$\beta : C_c^r(G, E_1) \times C_c^s(G, E_2) \to C_c^t(G, F), \qquad (f, g) \mapsto f *_b g$$

whith $(f *_b g)(x) := \int_G b(f(y), g(y^{-1}x))dy$. I'll characterize those $(G, r, s, t, b)$ for which $\beta$ is continuous. Surprisingly, the answer depends on $b$.

References:

[1] Birth, L., Untersuchungen zur Faltung auf Liegruppen," Diplomarbeit, University of Paderborn, November 2011.

[2] Birth, L. and H. Glöckner, Continuity of convolution of test functions on Lie groups, in preparation.

[3] Glöckner, H., Continuity of bilinear maps on direct sums of topological vector spaces, preprint, arXiv:1108.0169v2.

[4] Hirai, T., H. Shimomura, N. Tatsuuma, E. Hirai, Inductive limits of topologies, their direct product, and problems related to algebraic structures, J. Math. Kyoto Univ. 41:3 (2001), 475505.


## Frayday, February 10.

9h00-10h00. Valentin OVSIENKO. "*Alternated Hochschild cohomology and Lie antialgebras*".

Abstract: I will construct a graded Lie algebra on the space of cochains on a $Z_2$-graded vector space skew-symmetric in the odd variables. The Lie bracket is obtained from the classical Gerstenhaber bracket by (partial) skew-symmetrization; the coboundary operator is an amazing mixture of the Hochschild and Chevalley-Eilenberg differentials. I will show that an order-one element $m$ satisfying the zero-square condition defines an algebraic structure called "Lie antialgebra", I will explain geometric origins of these algebras. Two examples of non-trivial cohomology classes

will be constructed, these classes are similar to the celebrated Gelfand-Fuchs and Godbillon-Vey classes.

10h15-11h15. Julia KUZNETZOVA. "*A duality of locally compact groups independent of the Haar measure.*"

Abstract: We present a duality construction for locally compacts groups that is simpler than the theory of Kac algebras and does not involve the Haar measure in the definition of the duality functor. On the category of coinvolutive Hopf-von Neumann algebras (roughly speaking, these are Kac algebras without weight), we define a functor $M \mapsto \widehat{M}$ such that for every locally compact group $G$, the algebra $C_0(G)^{**}$ is reflexive. Here $C_0(G)^{**}$ is the enveloping von Neumann algebra of $C_0(G)$, identified with its second dual. The dual algebra $\widehat{C_0(G)^{**}}$ is the big group algebra $W^*(G)$ of J.Ernest. Moreover, we prove that for every coinvolutive Hopf-von Neumann algebra $M$, its second dual $\hat{\hat{M}}$ is reflexive, i.e. $\hat{\hat{M}} \simeq (\hat{\hat{M}})\hat{\ }$.

10h15-11h15. Gert HECKMAN. "*On the regularization of the Kepler problem.*"

Abstract: After a review of various classical solutions of the Kepler problem we discuss the regularization of the Kepler problem. We will show that the canonical regularization of Ligon and Schaaf from 1976, obtained by elaborate calculations (even in the improved form of Cushman and Duistermaat), can be derived as a straightforward consequence of the regularization of Moser from 1970. This is joint work with Tim de Laat.

14h00-15h00. Bernhard KELLER, "*On cluster monomials (joint work with G. Cerulli, D. Labardini and P.-G. Plamondon)*".

Abstract: According to Fomin-Zelevinsky's philosophy, each cluster algebra should admit a "canonical" basis, which should contain the cluster monomials. This lead them to formulate, about ten years ago, the conjecture on the linear independence of the cluster monomials. We will sketch a proof valid for all cluster algebras associated with quivers.

15h00-16h00. Stphane GAUSSENT, "*Hovels and Spherical Hecke algebras for Kac-Moody groups over local fields*".

Abstract: I will report on a joint work with Guy Rousseau. Let $G$ be a Kac-Moody group over a local field $F$. In a former work, we have defined a generalisation of the Bruhat-Tits building associated to $G$, called an hovel. The group $G$ acts upon it as a reductive group acts upon the building. But in a hovel two points are not always in

a same apartment. Nevertheless, the stabiliser $K$ of zero in the standard apartment plays the rôle of a maximal open compact subgroup and we can define the algebra of $K$-biinvariant functions on $G$ (with almost finite support), the spherical Hecke algebra associated to $(G, K)$.

16h15-17h15. A. ALLDRIDGE, " *Superbosonisation, Riesz superdistributions and unitary highest weight representations of Hermitian supergroups*".

Abstract: We report on joint work with our post-doc Zain Shaikh. Superbosonisation, pioneered by Littelmann–Sommers–Zirnbauer (2009), is a new technique to transform integrals to Berezin integrals in the framework of SUSY $\sigma$-models. We connect it to superorbit geometry in a certain super-Grassmannian, to Riesz superdistributions, and to unitary highest weight representations of Hermitian supergroups. We give two new proofs of the superbosonisation formula, one using the Laplace transform of tempered superdistributions, and one based on representation theory.

## Saturday, February 11.

9h00-10h00. Alexander PREMET. "*Completely prime primitive ideals with a prescribed associated variety*".

Abstract: An old problem of Lie Theory asks whether for any nilpotent orbit $O$ in a finite dimensional simple Lie algebra $\mathfrak{g}$ over complex numbers there exists a completely prime primitive ideal in the enveloping algebra $\mathcal{U}(\mathfrak{g})$ whose associated variety coincides with the Zariski closure of $O$. (A two-sided ideal $I$ of $\mathcal{U}(\mathfrak{g})$ is called completely prime if the quotent $\mathcal{U}(\mathfrak{g})/I$ is a domain). In my talk I'm going to discuss the current status of this problem.

10h00-11h00 Fanny KASSEL, "*Discrete spectrum for non-Riemannian locally symmetric spaces*".

Abstract: I will discuss spectral analysis of the Laplacian on certain pseudo-Riemannian manifolds of the form $X/\Gamma$, where $X$ is a reductive symmetric space and $\Gamma$ a discrete group of isometries acting properly discontinuously. When $X$ is non Riemannian, the Laplacian is not an elliptic operator anymore. In joint work with Toshiyuki Kobayashi, we prove that for a large class of such manifolds, the discrete spectrum of the Laplacian is infinite and stable under small deformations of $\Gamma$.

11h15-12h15 Kenji IOHARA, "*On $Z^2$-graded simple Lie algebras* ".

Abstract: Let $\Lambda$ be a lattice of rank $n$ and $\mathfrak{g}$ be a $\Lambda$-graded Lie algebra. For $n = 1$, the classification of $\Lambda$-graded simple Lie algebras with finite growth has been achieved by O. Mathieu. In this talk, I will present our recent result with O. Mathieu in the case of $n = 2$ assuming multiplicity freeness. I will also explain its relation with another domain.