PROGRAM

	Monday	Tuesday	Wednesday	Thursday	Friday
9h-9h50	***Registration***	E. Vinberg	D.Leites-I.Schepochkina	Yu. Neretin	A. Sossinsky
10h-10h50	G. Olshanski	D. Gourevitch	A. Braverman	P.E. Paradan	D. Millionschikov
	Coffee Break				
11h20-12h10	A. Okounkov	A. Panov	P. Etingof	A. Lachowska	M. Nazarov
	Lunch			***Lunch***	
14h-14h50	A. Vershik	A. Alekseev		M. Kontsevich	D. Fuchs
15h-15h50	J. Bernstein	A. Moreau	**Ceremony**	T. Kobayashi	A. Kirillov
	Coffee Break		***Dr Honoris Causa***	***Coffee Break***	
16h30-17h20	A. Sergeev	A. Molev		A. Karabegov	
	Wine Party			***Dinner***	

Talks	Amphi 2	***Coffee break***	Salle de séminaire	***Lunch***	RU – CROUS
Poster Session	Département MMI	***Wine Party***	Salle 3-R08	***Dinner***	Brasserie Flo

REPRESENTATION THEORY AT THE CROSSROADS OF MODERN MATHEMATICS IN HONOR OF ALEXANDRE KIRILLOV

REIMS, MAY 29 - JUNE 2, 2017

A. Alekseev. On non-commutative Kirillov–Kostant–Souriau brackets. The following theorem is one of the cornerstones of symplectic geometry: let G be a connected Lie group and M be a symplectic homogeneous space of G. Then, M is symplectomorphic to (a cover of) a coadjoint orbit of G with Kirillov symplectic form. This is one of the rare instances when a moment map uniquely determines the symplectic (or Poisson) structure.

It turns out that in the non-commutative Poisson geometry à la Kontsevich the situation is different: the theorem by F. Naef states that a Poisson structure with a given moment map is always unique (under some technical assumptions). Examples of such structures are given by non-commutative Kirillov–Kostant–Souriau (KKS) brackets. The corresponding Lie algebra of functions can be upgraded to a Lie bialgebra with Schedler cobracket. If time permits, I'll explain that the Grothendieck-Teichmueller Lie algebra acts by derivations of this Lie bialgebra structure.

J. Bernstein. A remark on the Satake isomorphism and Modified Langlands' Dual Group. Let G be a connected reductive group defined over a p-adic field F. In 1970-th Langlands proposed a scheme how to describe irreducible representations of G. He suggested that this can be done in terms of the dual group $\check{G}$. One of his motivation and tools was the Satake transform.

The aim of my lecture is to explain that there is no canonical Satake transform related to the group $\check{G}$. I will explain how to define a canonical Satake transform with values in a modifed Langlands' dual group that should be used in Langlands' approach.

(In fact some version of this group already has been introduced by Buzzard and Gee but my presentation will be slightly different.)

A. Breverman. Spectral description of the asymptotic Hecke algebra (joint work with D.Kazhdan). Let G be a split reductive p-adic group. Let $H(G)$ be its Hecke algebra and let $H(G,I)$ be its Iwahori part, which is known to be isomorphic to the corresponding affine Hecke algebra. In the 80's Lusztig using certain construction involving Kazhdan–Lusztig basis introduced certain algebra J (which is often called the asymptotic Hecke algebra) which contains $H(G,I)$ as a subalgebra (and this inclusion becomes an isomorphism after certain completion) which has many remarkable properties (I will recall some of them in my talk). The purpose of this talk is to give a representation theoretic description of J which in particular implies that

1) J is naturally a subalgebra of the Harish–Chandra Schwartz space $C(G)$
2) One can give a definition of J for the full $H(G)$ (*i.e.* not just the Iwahori component).

Some applications of this construction will be discussed.

P. Etingof. A counterexample to the Poincaré-Birkhoff-Witt theorem. I will give a counterexample to the Poincaré-Birkhoff-Witt theorem in characteristic p (don't get too excited - in the Verlinde category, which has no realization in vector spaces), and then explain how to correct this theorem. For this we will need to develop a theory of Koszul duality in symmetric tensor categories.

D. Fuchs. Schubert Varieties and their shadows.

D. Gurevich. Cayley-Hamilton Identity in quantum algebras. I plan to exhibit different forms of the CH identity in some quantum algebras. In particular, I'll discuss Kirillov's "family algebras" and their possible generalizations. Applications of the CH identity to integrable systems theory will be discussed as well.

A. Karabegov. A heat kernel proof of the algebraic index theorem. We give a heat kernel proof of the algebraic index theorem for the deformation quantizations with separation of variables on a pseudo-Kaehler manifold.

A.A. Kirillov. Representations of the triangular group over a finite field. I discuss some new phenomena, arising in the application of the orbit method to finite groups.

T. Kobayashi. Analysis of minimal representations — an approach to quantize nilpotent orbits. Minimal representations are the "smallest" infinite dimensional unitary representations of reductive groups. I plan to discuss geometric analysis of minimal representations from the viewpoint of orbit philosophy.

M. Kontsevich. Algebra of topological recursion. 10 years ago B. Eynard and N. Orantin proposed a remarkable formalism of topological recursion with numerous applications to mathematical physics: matrix models, string partition functions, knot invariants, WKB solutions,... One of features of this formalism is that it produces recursively certain expressions which are not symmetric on the nose, and the symmetry is proven by an obscure inductive argument involving multi-dimensional residues. I'll explain a simple algebraic mechanism behind this symmetry (joint work with Y. Soibelman). Namely, any Lagrangian manifold in a symplectic affine space given by a system of at most quadratic equations closed under the Poisson bracket, is canonically quantized.

A. Lachowska. Small quantum groups and diagonal coinvariants in type A. The question of the dimension of the center of the small quantum group at a root of unity has been open since the object was defined by Lusztig in 1990. The existing methods did not provide an answer for rank higher than 1. Starting from the description of the blocks of the small quantum group via an equivalence with a category of sheaves over the Springer resolution, we develop an algorithmic method for calculating the dimension of the regular and singular blocks of the center. The answers obtained in cases A_2 and A_3 allow us to put forward a conjecture relating the center of the small quantum group in type A_n with Haiman's diagonal coinvariant algebra for the symmetric group S_{n+1}. This is a joint work with Qi You (Yale/Caltech).

D. Leites and I. Shchepochkina (joint work with S. Bouarroudj and A. Lebedev). On classification of simple Lie superalgebras in characteristic 2. In late 1960s, Kostrikin and Shafarevich suggested a method producing all finite-dimensional simple Lie algebras over an algebraically closed field of characteristic $p > 7$, and sketched a conjectural list thus obtained. The method proved correct for $p > 5$, and even for $p > 3$ (with an amendment). For restricted algebras for $p > 5$, the conjecture was proved (Block and Wilson) in 1988, in general about 2008 (mainly Premet and Strade). The $p = 3$ case brings more examples (conjecturally, all $\mathbb{Z}$-graded ones are known), the $p = 2$ case seems (and is) terribly complicated.

Classification of simple Lie superalgebras is considerably more difficult than the classification of simple Lie algebras, even in characteristic 0, and the difficulties pile up, as $p > 0$ is getting small (..., 5, 3), and exacerbate the problem.

A hasty opinion that there is no difference between Lie algebras and Lie superalgebras if $p = 2$ is completely wrong. The latter are defined by squarings of odd elements, not brackets, and we accordingly modify the Jacobi identity and the notion of derived algebra.

Therefore it is a totally unexpected lucky break that for $p = 2$ there are two simple methods that from each simple Lie algebra produce simple Lie superalgebras and all simple Lie superalgebras are obtained via these methods.

D. Millionschikov. Slow-growing Lie algebras. The growth of a finitely generated infinite-dimensional Lie algebra $\mathfrak{g}$ can be described by the Gelfand-Kirillov dimension which is defined as

$$GK \dim \mathfrak{g} = \limsup_{n\to\infty} \frac{\log \dim V^n}{\log n},$$

where V^n is the subspace in $\mathfrak{g}$ spanned by all elements of length at most n with arbitrary arrangements of brackets. A finite Gelfand–Kirillov dimension means that there exists a polynomial $P(x)$ such that $\dim V^n < P(n)$ for all $n>1$. Shalev and Zelmanov obtained [5] important results on Lie algebras of GK-dimension 1, i.e on Lie algebras of linear growth. Within this class, they have distinguished a special subclass of the positively graded two-generated Lie algebras with the slowest possible growth ($\dim V^n = n+1, n>1$), the so-called Lie algebras of maximal class. For instance the positive part W^+ of the Witt (Virasoro) algebra is an example of a Lie algebra of maximal class.

Kac [2] classified under a certain technical condition infinite-dimensional $\mathbb{Z}$-graded simple Lie algebras $\mathfrak{g} = \oplus_{i\in\mathbb{Z}}\mathfrak{g}_i$ of finite growth in a following sense: $\dim \mathfrak{g}_n \leq P(n)$ for some polynomial $P(x)$. Moreover, Kac conjectured that dropping the condition would add only the Witt algebra. Finally Kac's conjecture was proved in 1990 by Mathieu [3].

Fialowski [1] classified $\mathbb{N}$-graded Lie algebras $\mathfrak{g} = \oplus_{i\in\mathbb{N}}\mathfrak{g}_i$ with one-dimensional homogeneous components $\mathfrak{g}_i$ that are multiplicatively generated by two elements from $\mathfrak{g}_1$ and $\mathfrak{g}_2$ respectively. Besides other algebras one can find in her list W^+ and two positively graded Lie algebras $\mathfrak{n}_1$ and $\mathfrak{n}_2$ that are maximal nilpotent subalgebras of twisted loop algebras $A_1^{(1)}$ and $A_2^{(2)}$ respectively.

A Lie algebra $\mathfrak{g}$ is called naturally graded if it is isomorphic to $\mathrm{gr}_C\mathfrak{g}$, its associated graded Lie algebra with respect to the filtration by ideals $C^i\mathfrak{g}$ of the descending central sequence. W^+ is not naturally graded, because $\mathrm{gr}_C W^+ \cong \mathfrak{m}_0$, where $\mathfrak{m}_0$ can be defined by its infinite basis $e_1, e_2, \dots$, and structure relations: $[e_1, e_i] = e_{i+1}, i = 2, 3, \dots$ $[e_k, e_l] = 0, k, l \geq 2$. It was proved by Vergne [6] that up to an isomorphism there is the only one naturally graded Lie algebra of maximal class and it is $\mathfrak{m}_0$.

We classify two-generated naturally graded Lie algebras $\mathfrak{g} = \oplus_{i\in\mathbb{N}}\mathfrak{g}_i$ with the following linear growth:

$$\dim V^n \leq \frac{3}{2}n+\frac{1}{2}.$$

An interesting feature of this classification is the difference between the real and complex cases.

References

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[5] Shalev A., Zelmanov E.I. Narrow algebras and groups. J. of Math. Sciences. 1999. V. 93, P. 951–963.

[6] Vergne M. Cohomologie des algèbres de Lie nilpotentes. Bull. Soc. Math. France. 1970. V. 98. P. 81–116.

A. Molev. Quantization of the shift of argument subalgebras. Given a simple Lie algebra $\mathfrak{g}$ and an element $\mu \in \mathfrak{g}^*$, the corresponding shift of argument subalgebra of $S(\mathfrak{g})$ is Poisson commutative. In the case where μ is regular, this subalgebra is known to admit a quantization, that is, it can be lifted to a commutative subalgebra of $U(\mathfrak{g})$. We show that if $\mathfrak{g}$ is of type A, then this property extends to arbitrary μ, thus proving a conjecture of Feigin, Frenkel and Toledano Laredo. The proof relies on an explicit construction of generators of the center of the affine vertex algebra at the critical level.

A. Moreau. Sheet and associated varieties of vertex algebras. To any vertex algebra, one can associate a certain Poisson variety, called the associated variety. Contrary to the associated varieties of primitive ideals in the enveloping algebra, the associated variety of affine vertex algebras no need to be contained in the nilpotent cone. We will see that sheet closures appear in this context, and will discuss some conjectures and open problems on the irreducibility of these associated varieties. The talk will be based on joint works with Tomoyuki Arakawa.

M. Nazarov. Cherednik operators at infinity. Heckman introduced N operators on the space of polynomials in N variables, such that these operators form a covariant set relative to permutations of the operators and variables, and such that Jack symmetric polynomials are eigenfunctions of the power sums of these operators. We introduce the analogues of these N operators for Macdonald symmetric polynomials, by using Cherednik operators. The latter operators pairwise commute, and Macdonald polynomials are eigenfunctions of their power sums. We compute the limits of our operators at $N \to \infty$. These limits yield a Lax operator for Macdonald symmetric functions. This is a joint work with Evgeny Sklyanin.

Yu. Neretin. Multiplication of double cosets on infinite symmetric group and combinatorial cobordisms. Consider a triple product G of infinite symmetric groups, the diagonal K, and subgroups $K(n)$ in K fixing first n points. We describe double cosets $K(n) \setminus G/K(m)$ in terms of two-dimensional surfaces with special colored triangulations. We show that they form a category (multiplication is similar to concatenation of cobordisms) and unitary representations of G produce representations of this category.

A. Okounkov. Enumerative geometry and geometric representation theory. Certain problems in enumerative geometry turn out to be directly related to core questions in representation theory and to how it has been classically applied in mathematical physics. In my talk, I plan to give an informal introduction to this circle of ideas.

G. Olshanski. Quantization of harmonic analysis on U^∞ and applications. I will report on recent results based on the joint work with Vadim Gorin (*J. Funct. Anal.* 2016) and its further development.

A.N. Panov. Supercharacter theory and the orbit method. Given a finite group G, suppose that we have two partitions of equal number of components $\mathrm{Irr}(G) = X_1 \cup \dots \cup X_m$ and $G = K_1 \cup \dots \cup K_m$. These partitions form a *supercharacter theory* if for any X_j the character $\sigma_i = \sum_{\psi \in X_i} \psi(1)\psi$ is constant on each K_j. The characters $\{\sigma_i\}$ are called *supercharacters* and K_j – *superclasses*. The quadratic table $(\sigma_i(K_j))$ is called a *supercharacter table*.

An example of supercharacter theory is the theory of irreducible characters (each X_i is a singleton subset and K_j is a class of conjugate elements). For some groups, like the unitriangular group $\mathrm{UT}(n, \mathbb{F}_q)$, classification of all irreducible characters is a "wild" problem. In this case, the main goal is to construct a supercharacter theory as a "first approximation" of the theory of irreducible characters. Such supercharacter theories were constructed for the unitiangular group by C. Andre and for the algebra groups by P.Diaconis and I.Isaacs [1].

We present the superchararacter theory for the finite groups of triangular type [2]. We concern restriction and superinduction of supercharcaters, obtain the analog of the A.A.Kirillov formula for supercharacters and characterize the Hopf algebra of supercharacters for the triangular group $\mathrm{T}(n, \mathbb{F}_q)$.

References

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P.E. Paradan. Kirillov's orbits method: the case of the discrete series. In Kirillov's orbits method, one of the important points is the description of the restriction of a representation $V(O)$ associated to a coadjoint orbit O of a group G to a subgroup H. The purpose of our talk is to explain how to achieve this goal when $V(O)$ is a representation of the discrete series of G and H is a reductive subgroup of G such that the restriction of $V(O)$ to H is admissible. We will show that the H-multiplicities which appear can be expressed geometrically by means of the symplectic reductions of O relatively to the action of H.

A. Sergeev. Super Jack-Laurent polynomials. Let $D_{n,m}$ be the algebra of the quantum integrals of the deformed Calogero-Moser-Satherland problem corresponding to the root system of the Lie superalgebra $gl(n, m)$. The algebra $D_{n,m}$ acts naturally on the quasi-invariant Laurent polynomials and we investigate the corresponding spectral decomposition. Even for general value of the parameters the spectral decomposition is not simple and we prove that every generalised eigenspace has dimension 2^r for some natural r and we also prove that the action of the algebra $D_{n,m}$ in the generalized eigenspce is the same as the regular representation of the algebra $k[\varepsilon]^{\otimes r}$, where $k[\varepsilon]$ is the algebra of the dual numbers.

A.B. Sossinsky. Configuration spaces of planar linkages. In the early1990-ies, returning from short stay in the US, a very exited Sasha Kirillov told me about a lecture of Thurston that he had attended there. The lecture was about planar linkages (*aka* hinge mechanisms, шарнирные механизмы, systèmes articulés, Gelenkenmechanismen). Retold to me from memory by Sasha, it excited me even more, I tried to fill in the missing proofs (not always succeeding) and started to lecture on the subject to anyone who would listen, from high school students to experts in Teichmuller theory. In a brief review of what happened after that, I will give as many examples of results in this beautiful field as time permits. They should include:

- Classification of configuration spaces of generic pentagons [Dmitry Zvonkin].
- Configuration spaces of spiders [Anna Kondakova].
- Morse index of cyclic polygons [G. Panina, A. Zhukova].
- Configuration spaces of linkages and a signature formula [G. Khimshiashvili].

A.M. Vershik. Improbable properties of classical Bernoulli scheme discovered by the litlle regular representation of infinite symmetric group. A remarkable recent continuation of the old ('87) paper by Kerov & Vershik (on RSK) given by Piotr Sniady ('14) allows to start a new stage of theory of representations across probability.

1) RSK classical and RSK infinite (KV). Forgotten question.
2) Homomorphism to the space of right tableau is isomorphism (Sniady).
3) New representation of S_{infty} in L^2 over classical Bernoulli scheme.
4) Monotonic structure defines everything; Jeu de taquin and two-sided problem for Young diagrams.

E.B. Vinberg. Non-abelian gradings of Lie algebras. Let S be a reductive (complex) algebraic group. An S*-structure* in a Lie algebra $\mathfrak{g}$ is a homomorphism $\Phi : S \to \mathrm{Aut}\mathfrak{g}$. If the group S is abelian, then the weight subspaces of $\mathfrak{g}$ with respect to S constitute a grading of the algebra $\mathfrak{g}$. In particular, the root decompositions of semisimple Lie algebras, various cyclic gradings etc. are obtained in this way.

If S is not abelian, the isotypic decomposition of $\mathfrak{g}$ with respect to S can be viewed as a "non-abelian grading". In general, the commutation operation in $\mathfrak{g}$ does not admit a reasonable description in terms of such grading. However, in some simplest cases it admits a simple description, which provides, in particular, some interesting models of the exceptional simple Lie algebras.

We will consider two types of S-structures: very short SO_3-structures and short SL_3-structures, where SO_3 and SL_3 stand for $\mathrm{SO}_3(\mathbb{C})$ and $\mathrm{SL}_3(\mathbb{C})$ for brevity.

A non-trivial SO_3-structure is called *very short*, if the representation Φ decomposes into 1- and 3-dimensional irreducible representations. A very short SO_3-structure in a simple

Lie algebra $\mathfrak{g}$ gives rise to the isotypic decomposition

$$\mathfrak{g} = \mathrm{Der} J + \mathfrak{so}_3 \otimes J \quad (\simeq \mathrm{Der} J + \mathfrak{sl}_2 \otimes J),$$

where J is a simple Jordan algebra. The commutator of two elements of the second summand is determined by the formula

$$[a \otimes x, b \otimes y] = (a, b)\Delta(x, y) + [a, b] \otimes xy,$$

where $\Delta(x, y)$ is an inner derivation of J (given by some explicit formula). Conversely, any simple Jordan algebra defines a simple Lie algebra by the above formulas. This connection between Lie and Jordan algebras goes back to J. Tits, I. Kantor, and M. Koecher.

Among the exceptional simple Lie algebras, only E_7 admits a very short SO_3-structure. It corresponds to the Jordan algebra of Hermitian (3×3)-matrices over (complex) octonions (the Albert algebra).

A non-trivial SL_3-structure is called *short*, if the representation Φ decomposes into the adjoint representation of SL_3 and 1- and 3-dimensional irreducible representations. In each simple Lie algebra but C_n there exists a short SL_3-structure, and it is unique up to conjugation. For $\mathfrak{g} = A_n$, the corresponding isotypic decomposition is essentially a $\mathbb{Z}$-grading of depth 1. In all the other cases, the isotypic decomposition has the form

$$\mathfrak{g} = \mathfrak{s} + (\mathrm{Der} J + J_0) + V \otimes J + V^* \otimes J,$$

where $\mathfrak{s} = d\Phi(\mathfrak{sl}_3) \simeq \mathfrak{sl}_3$, V is the space of the tautological representation of $\mathfrak{sl}_3$, J is a semisimple Jordan algebra of rank ≤ 3, and $J_0 \subset J$ is the subspace of elements of trace 0.

The algebra $\mathfrak{g}$ is uniquely reconstructed from J. In particular, in this way the non-trivial one-dimensional Jordan algebra corresponds to the Lie algebra G_2, and the Jordan algebras of Hermitian (3×3)-matrices over the four composition algebras correspond to the Lie algebras F_4, E_6, E_7, and E_8.